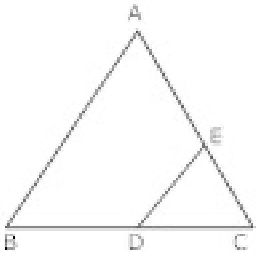
CBSE Class 10 Mathematics
Important Questions
Chapter 6
Triangles

1 Marks Questions

1. In the figure $\Delta ABC \sim \Delta EDC$, if we have AB = 4 cm, ED = 3 cm, CE = 4.2 cm and CD = 4.8 cm, then the values of CA and CB are



(a) 6 cm, 6.4 cm

(b) 4.8 cm, 6.4 cm

(c) 5.4 cm, 6.4 cm

(d) 5.6 cm, 6.4 cm

Ans. (d) 5.6 cm, 6.4 cm

2. The areas of two similar triangles are respectively $9\ cm^2$ and $16\ cm^2$. Then ratio of the corresponding sides are

(a) 3:4

(b) 4:3

(c) 2:3

(d) 4:5

Ans. d) 4:5

3. Two isosceles triangles have equal angles and their areas are in the ratio 16:25, then the ratio of their corresponding heights is

(a) $\frac{4}{5}$

(b) $\frac{5}{4}$

(c) $\frac{3}{6}$

(d) $\frac{5}{7}$

Ans. (a) $\frac{4}{5}$

4. If $\Delta ABC \sim \Delta DEF$ **and** $AB = 5\ cm,$ **area** $(\Delta ABC) = 20\ cm^2,$ **area** $(\Delta DEF) = 45\ cm^2,$ **then DE =**

(a) $\frac{4}{5}$ **cm**

(b) 7.5 cm

(c) 8.5 cm

(d) 7.2 cm

Ans. (b) 7.5 cm

5. A man goes 15 m due west and then 8 m due north. Find distance from the starting point.

(A) 17 m

(B) 18 m

(C) 16 m

(D) 7 m

Ans. (A) 17 m

6. In a triangle ABC, if AB = 12 cm, BC = 16 cm, CA = 20 cm, then ΔABC is

(A) Acute angled

(b) Right angled

(c) Isosceles triangle

(d) equilateral triangle

Ans. (b) Right angled

7. In an isosceles triangle ABC, AB=AC=25 cm and BC = 14 cm, then altitude from A on BC =

(a) 20 cm

(b) 24 cm

(c) 12 cm

(d) None of these

Ans. (b) 24 cm

8. The side of square who's diagonal is 16 cm is

(a) 16 cm

(b) $8\sqrt{2}\ cm$

(c) $5\sqrt{2}\ cm$

(d) None of these

Ans. (b) $8\sqrt{2}\ cm$

9. In an isosceles triangle ABC, if AC = BC and $AB^2 = 2AC^2$, then $\angle C =$

(a) $45°$

(b) $60°$

(c) $90°$

(d) $30°$

Ans. (c) $90°$

10. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) $BC \times EF = AC \times FD$

(b) $AB \times EF = AC \times DE$

(c) $BC \times DE = AB \times EF$

(d) $BC \times DE = AB \times FD$

Ans. c) $BC \times DE = AB \times EF$

11. A certain right-angled triangle has its area numerically equal to its perimeter. The length of each side is an even integer, what is the perimeter?

(a) 24 units

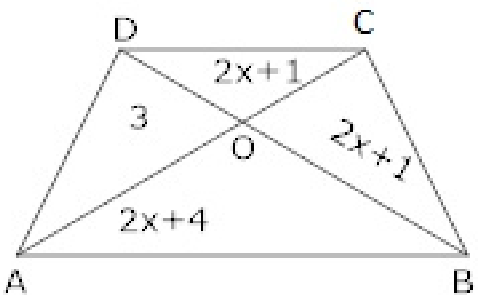
(b) 36 units

(c) 32 units

(d) 30 units

Ans. (a) 24 units

12. In the given figure, if AB | | CD, then $x =$



(a) 3

(b) 4

(c) 5

(d) 6

Ans. (a) 3

13. Length of an altitude of an equilateral triangle of side '2a' cm is

(a) 3a cm

(b) $\sqrt{3}a\ cm$

(c) $\frac{\sqrt{3}}{2}a\ cm$

(d) $2\sqrt{3}a\ cm$

Ans. (b) $\sqrt{3}a\ cm$

14. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

(a) $\Delta PQR \sim \Delta CAB$

(b) $\Delta PQR \sim \Delta ABC$

(c) $\Delta CBA \sim \Delta PQR$

(d) $\Delta BCA \sim \Delta PQR$

Ans. a) $\Delta PQR \sim \Delta CAB$

15. The area of two similar triangles are $81\ cm^2$ and $49\ cm^2$ respectively. If the altitude of the bigger triangle is 4.5 cm, then the corresponding altitude of the smaller triangle is

(a) 2.5 cm

(b) 2.8 cm

(c) 3.5 cm

(d) 3.7 cm

Ans. c) 3.5 cm

16. In a right-angled triangle, if base and perpendicular are respectively 36015 cm and 48020 cm, then the hypotenuse is

(a) 69125 cm

(b) 60025 cm

(c) 391025 cm

(d) 60125 cm

Ans. (b) 60025 cm

17. In figure, DE| |BC and AD =1 cm, BD = 2 m. The ratio of the area of ΔABC to the area

of ΔADE is

(a) 9:1

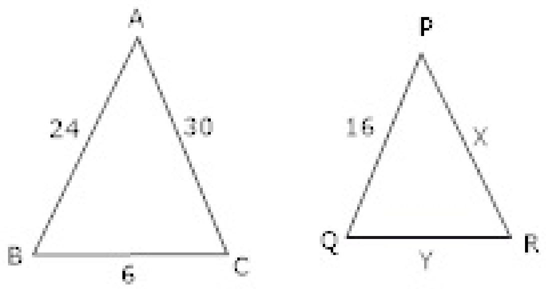
(b) 1:9

(c) 3:1

(d) none of these

Ans. (a) 9:1

18. In the given figure, $\Delta ABC \sim \Delta PQR$, then the value of x and y are



(a) $(x, y) = (6, 20)$

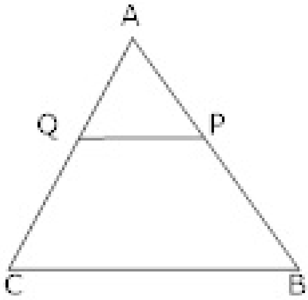
(b) $(20, 60)$

(c) $(x, y) = (3, 10)$

(d) none of these

Ans. (b) $(20, 60)$

19. In figure, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, AQ = 3 cm and QC = 6 cm. If PQ = 4.5 cm, then BC is



(a) 12.5 cm

(b) 5.5 cm

(c) 13.5 cm

(d) none of these

Ans. c) 13.5 cm

20. D, E, F are the mid-points of the sides AB, BC, and CA respectively of ΔABC , then $\frac{ar(\Delta DEF)}{ar(\Delta ABC)}$ is

(a) 1:4

(b) 4:1

(c) 1:2

(d) none of these

Ans. (a) 1:4

CBSE Class 10 Mathematics

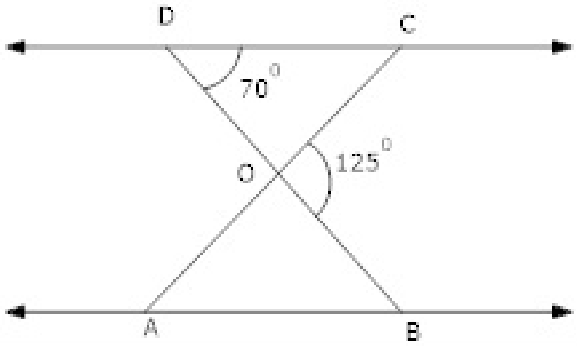
Important Questions

Chapter 6

Triangles

2 Marks Questions

1. In the given figures, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ **and** $\angle CDO = 70°$. **Find**



(i) $\angle DOC$

(ii) $\angle DCO$

(iii) $\angle OAB$

(iv) $\angle AOB$

(v) $\angle OBA$

Ans. (i) $\angle DOC = 180° - 125° = 55°$

(ii) $\angle DCO = 180° - (70° + 55°)$ [$\because$ DOB is a st. line and OC stands on it]

$= 180° - 125° = 55°$ [$\because$ sum of angles of a tringle $= 180°$]

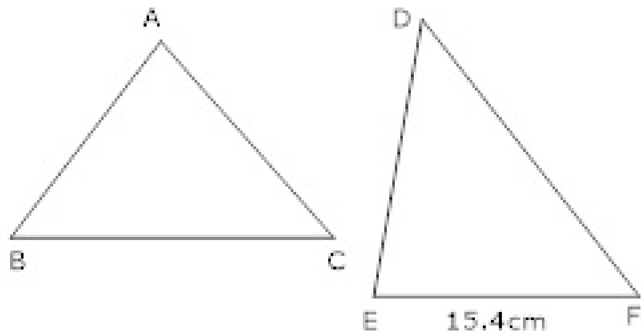
(iii) $\angle DAB = \angle DCO = 55°$

$\left[\begin{array}{l} \because \Delta ODC \sim OBA(\text{given}) \\ \therefore \angle DOC = \angle AOB, \angle ODC = \angle OBA, \angle DCO = \angle OAB \end{array}\right]$

(iv) $\angle AOB = \angle DOC = 55°$

(v) $\angle OBA = \angle ODC = 70°$

2. $\Delta ABC \sim \Delta DEF$ **and their areas are respectively 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.**



Ans. $Since\ \Delta ABC \sim \Delta DEF \therefore \dfrac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \dfrac{BC^2}{EF^2}$

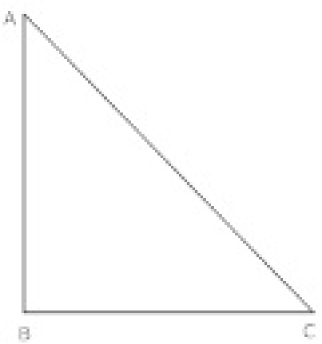
[$\because$ the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides]

$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow BC^2 = \frac{64 \times 154 \times 154}{121 \times 10 \times 10} = \frac{64 \times 14 \times 14}{100}$$

$$\Rightarrow BC = \frac{8 \times 14}{10} = 11.2 \text{ cm}$$

3. ABC is an isosceles right triangle right-angled at C. Prove that $AB^2 = 2AC^2$.



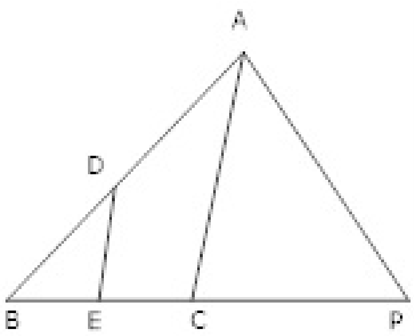
Ans. In right-angled $\Delta ABC, right \angle A\ at\ C$

$AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$= AC^2 + AC^2 = 2AC^2 \quad [\because BC = AC(given)]$

$= AB^2 = 2AC^2$

4. In the figure, DE||AC and $\frac{BE}{EC} = \frac{BC}{CP}$, prove that



Ans. In $\Delta ABC, DE \parallel AC$

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$.......$(i)$ [By Thales's Theorem]

Also $\frac{BE}{EC} = \frac{BC}{CP}(given)$.........$(ii)$

$\therefore$ from (i) and (ii), we get

$\frac{BD}{DA} = \frac{BC}{CP} \therefore DC \parallel AP$ [By the converse of Thales's Theorem]

5. The hypotenuse of a right triangle is 6 m more than the twice of the shortest side. If the third side is 2m less than the hypotenuse. Find the side of the triangle.

Ans. Let shortest side be $x\,m$ in length

Then hypotenuse $= (2x+6)m$

And third side = $(2x+4)m$

We have,

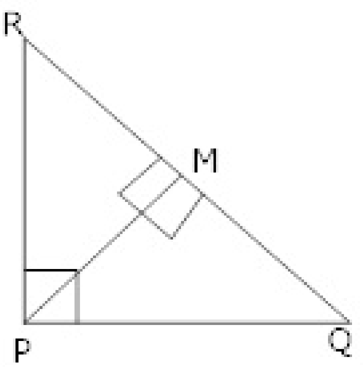
$$(2x+6)^2 = x^2 + (2x+4)^2$$
$$\Rightarrow 4x^2 + 24x + 36 = x^2 + 4x^2 + 16 + 16x$$
$$\Rightarrow x^2 - 8x - 20 = 0$$
$$\Rightarrow x = 10 \text{ or } x = -2$$

$$\Rightarrow x = 10$$

Hence, the sides of triangle are 10 m, 26 m and 24 m.

6. PQR is a right triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that $PM^2 = QM.MR$.

Ans. $\because PQR$ is a right triangle right angled at P and $PM \perp QR$



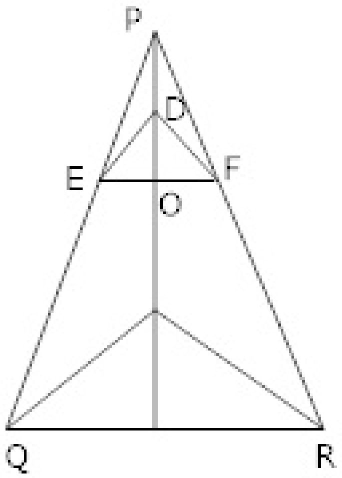
$\therefore \Delta PMR \sim \Delta PMQ$

$$\therefore \frac{PR}{PQ} = \frac{PM}{QM} = \frac{MR}{PM}$$

$$\Rightarrow \frac{PM}{QM} = \frac{MR}{PM}$$

i. e., $PM^2 = QM.MR$

7. In the given figure, $DE \| OQ$ **and** $DF \| OR$, **Prove that** $EF \| OQ$.



Ans. In ΔOQP, $DE \parallel OQ$

$$\frac{PE}{EQ}=\frac{PD}{DO} \text{..........}(i)$$

In ΔOPR, DF || OR

$$\frac{PD}{DO}=\frac{PF}{FR} \text{.........}(ii)$$

From (i) and (ii), we get

$$\frac{PE}{EQ}=\frac{PF}{FR}$$

$\therefore$ From ΔPQR,

$EF \parallel QR$

8. In figure, DE||BC, Find EC.

Ans. $\because DE \parallel BC$

$$\therefore \frac{AD}{DB}=\frac{AE}{EC}$$

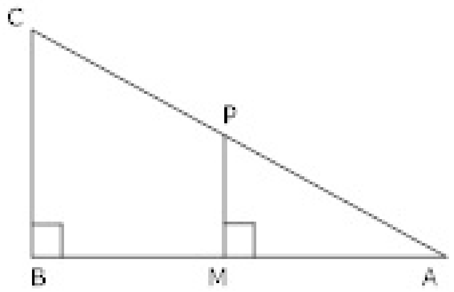
$$\Rightarrow \frac{1.5}{3}=\frac{1}{EC}$$

$\therefore EC = 2cm$

9. In the given figure, ABC and AMP are two right-angled triangles, right angled at B and M respectively, prove that

$(i)\ \Delta ABC \sim \Delta AMP$

$(ii)\ \frac{CA}{PA} = \frac{BC}{MP}$



Ans. In ΔABC and DAMP,

$\angle B = \angle M$ (Each 90°)

$\angle A = \angle A(\text{common})$

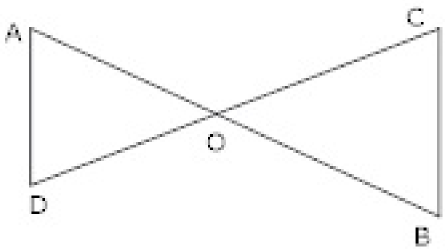
$\therefore \angle ACB = \angle APM$

$\therefore \Delta S$ are equiangular

i.e., $\Delta ABC \sim \Delta AMP$

$\therefore \frac{BC}{MP} = \frac{CA}{PA}$

10. In the given figure, OA × OB=OC × OD or $\frac{OA}{OC} = \frac{OD}{OB}$**, prove that** $\angle A = \angle C$ **and** $\angle B = \angle D$



Ans. In ΔAOD and ΔBOC,

$OA \times OB = OC \times OD$

i.e $\frac{OA}{OC} = \frac{OD}{OB}$

And $\angle AOD = \angle BOC$ [Vertically opposite Angles]

$\therefore \Delta AOD \sim \Delta BOC \left[By\ SAS\right]$

$\therefore \angle A = \angle C$ and $\angle B = \angle D$ [Corresponding angles of similar Δ]

11. In the given figure, DE||BC and AD=1 cm, BD = 2 cm. What is the ratio of the area of ΔABC to the area of ΔADE?

Ans. $\because DE \parallel BC$ in ΔABC

$\therefore \angle ADE = \angle ABC$

$\angle AED = \angle ACB$

Also $\angle DAE = \angle DAC$

$\therefore \Delta ADE \sim \Delta ABC$

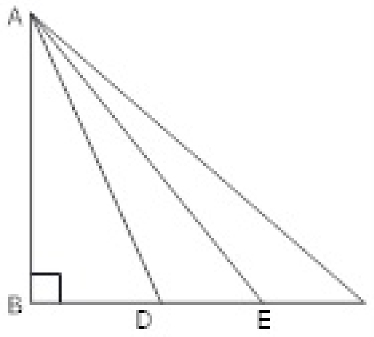
$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD^2}{AB^2} = \frac{area\left(\Delta ADE\right)}{area\left(\Delta ABC\right)}$

$\Rightarrow \frac{1^2}{3^2} = \frac{area\left(\Delta ADE\right)}{area\left(\Delta ABC\right)}\left[\because AB = AD + OB = 1 + 2 = 3\right.$

Hence, $\frac{area(\Delta ABC)}{area(\Delta ADE)} = \frac{9}{1}$

12. A right-angle triangle has hypotenuse of length p cm and one side of length q cm. If p – q =1, Find the length of third side of the triangle.



Ans. Let third side = x cm

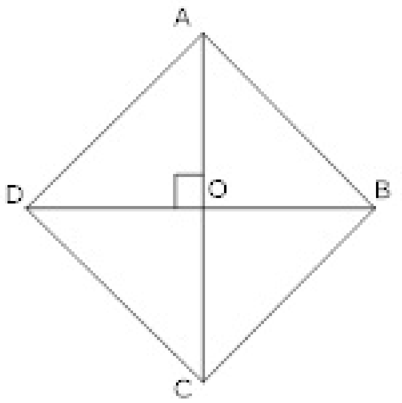
Then by Pythagoras theorem,

$$p^2 = q^2 + x^2$$
$$x^2 = p^2 - q^2$$
$$= (p+q)(p-q)$$
$$= (p+q) \times 1 \ (\because p-q=1)$$
$$= q+1+q$$
$$= 2q+1$$
$$\therefore x = \sqrt{2q+1}$$

13. The length of the diagonals of a rhombus are 24 cm and 10 cm. Find each side of rhombus.

Ans. $AC = 24\ cm \therefore AO = 12\ cm$

$BD = 10\ cm \therefore OD = 5\ cm$



From right-angled ΔAOD,

$$AD^2 = AO^2 + OD^2$$
$$\Rightarrow AD^2 = 12^2 + 5^2$$
$$\Rightarrow AD^2 = 169$$
$$\Rightarrow AD = 13\ cm$$

Hence each side = 13 cm

14. In an isosceles right-angled triangle, prove that hypotenuse is $\sqrt{2}$ times the side of a triangle.

Ans. Let hypotenuse of right-angled $\Delta = h$ units and equal sides of $triangle\ x$ units

$\therefore$ By Pythagoras theorem,

$$h^2 = x^2 + x^2$$
$$\Rightarrow h^2 = 2x^2$$
$$\Rightarrow h = \sqrt{2}x$$

15. In figure, express x in terms of a, b, c.

Ans. $\Delta ABO \sim \Delta OCD$

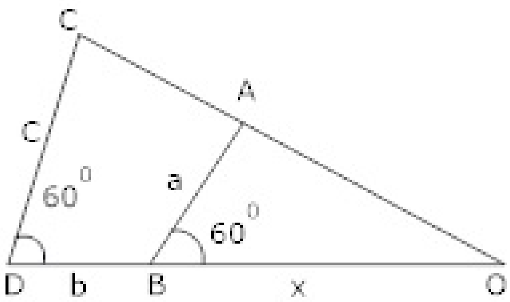
$$\Rightarrow \frac{x}{a} = \frac{x+b}{c}$$

$$\Rightarrow x = ax + ab$$

$$\Rightarrow x(c-a) = ab$$

$$\Rightarrow x = \frac{ab}{c-a}$$

16. The perimeter of two similar triangle ABC and PQR are respectively 36 cm and 24 cm. If PQ=10 cm, find AB.



Ans. $\Delta ABC \sim \Delta PQR$

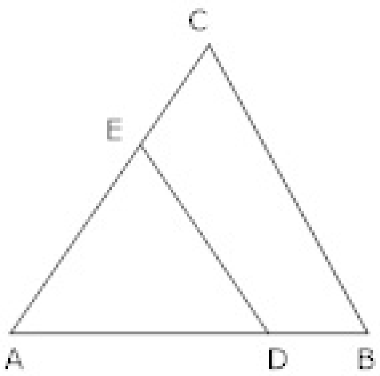
$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \frac{AB+BC+AC}{PQ+QR+PR} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR}$$

$$\Rightarrow \frac{AB}{10} = \frac{36}{24}$$

$$\Rightarrow AB = \frac{36\times 10}{24} = 15\ cm$$

17. In the given figure, DE||BC. If $AD = x, DB = x-2, AE = x+2, EC = x-1$ **find the value of** x.



Ans. In the given figure,

$$DE \parallel BC$$
$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$
$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$
$$\Rightarrow x^2 - x = x^2 - 4$$
$$\Rightarrow x = 4$$

18. The hypotenuse of a right-angled triangle is p cm and one of sides is q cm. if p = q+1, find the third side in terms of q.

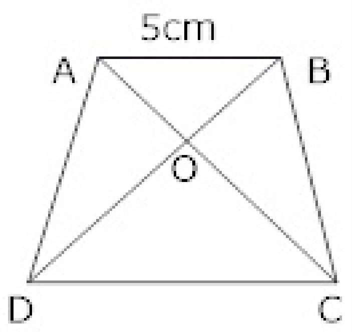
Ans. Let third side be $x\ cm$

$$\therefore p^2 = q^2 + x^2 \text{.........}(i)$$
$$\text{Also } p = q + 1 \text{.......}(ii)$$

From (i) and (ii), we get

$$(q+1)^2 = q^2 + x^2 \Rightarrow x^2 = 2q + 1$$
$$\Rightarrow x = \sqrt{2q+1}\ cm$$

19. In the given figure, $\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2}$ **and AB = 5 cm, find the value of DC.**



Ans. In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\frac{AO}{OC} = \frac{BO}{OD} \Rightarrow \frac{AO}{OB} = \frac{OC}{OD}$ [Given]

$\therefore \Delta AOB \sim \Delta COD$ [By SAS similarity]

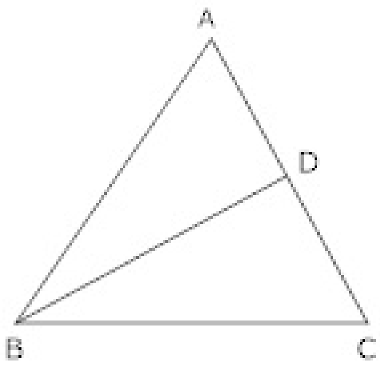
$\therefore \frac{AO}{CO} = \frac{BO}{DO} = \frac{AB}{CD}$

$\frac{1}{2} = \frac{AB}{DC}\left[\frac{AO}{OC} = \frac{BO}{OD} = \frac{1}{2} \text{ is given}\right]$

$\Rightarrow \frac{1}{2} = \frac{5}{DC}$

$\Rightarrow DC = 10\ cm$

20. In $\Delta ABC, AB = AC$ and D is a point on side AC, such that $BC^2 = AC \times CD$. Prove that BD = BC.



Ans. Given: A ΔABC in which AB = AC, D is a point on BC

To prove: BD = BC

Proof: $BC^2 = AC \times CD$ [given]

$$\Rightarrow \frac{BC}{AC} = \frac{DC}{BC}$$

In ΔABC and ΔBDC,

$$\Rightarrow \frac{BC}{CA} = \frac{DC}{CB} \text{ and } \angle C = \angle C \left[Common\right]$$

$\therefore \Delta ABC \sim \Delta BDC$ [SAS similarity]

$$\Rightarrow \frac{AB}{BD} = \frac{AC}{BC} \Rightarrow \frac{AC}{BD} = \frac{AC}{BC} \left[\because AB = AC\right]$$

$$\Rightarrow BD = BC$$

CBSE Class 10 Mathematics

Important Questions

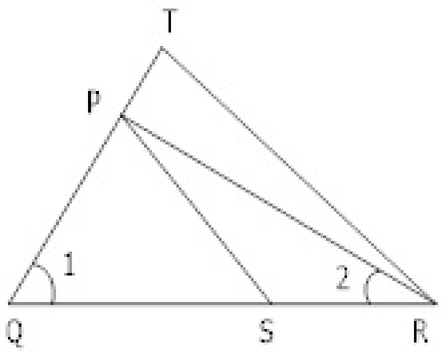
Chapter 6

Triangles

3 Marks Questions

1. In the given figure, $\frac{QT}{PR}=\frac{QR}{QS}$ and $\angle 1=\angle 2$. Prove that

$\Delta PQS \sim \Delta TQR$.



Ans. Since $\frac{QT}{PR}=\frac{QR}{QS}[Given]$

$\therefore \frac{QT}{QR}=\frac{PR}{QS}.......(i)$

Since $\angle 1=\angle 2$ [Given]

$PQ=PR........(ii)$

[In ΔPQR sides opposites to opposite angles are equal]

$\therefore \frac{QT}{QR}=\frac{PQ}{QS}........(iii)$ [Form(i)and(ii)]

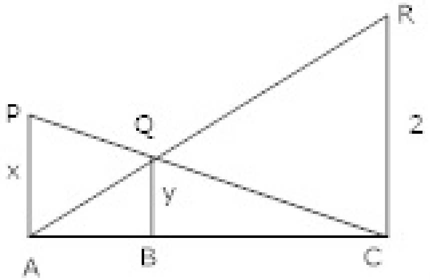
Now in ΔPQS and ΔTQR

From (iii), $\frac{PQ}{QS}=\frac{QT}{QR}$ i.e. $\frac{PQ}{QT}=\frac{QS}{QR}$

And $\angle Q=\angle Q$ [Common]

$\therefore \Delta PQS \sim \Delta TQR$ [By S.A.S. Rule of similarity]

2. In the given figure, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x}+\frac{1}{2}=\frac{1}{y}$.



Ans. In ΔPAC and ΔQBC,

$\angle PAC = \angle QBC$ [Each $= 90°$]

$\angle PCA = \angle QCB$ [Common]

$\therefore \Delta PAC \sim \Delta QBC$

$\frac{x}{y} = \frac{AC}{BC}$ i.e. $\frac{y}{x} = \frac{BC}{AC}$ ....(i)

Similarly, $\frac{z}{y} = \frac{AC}{AB}$ i.e. $\frac{y}{z} = \frac{AB}{AC}$ ....(ii)

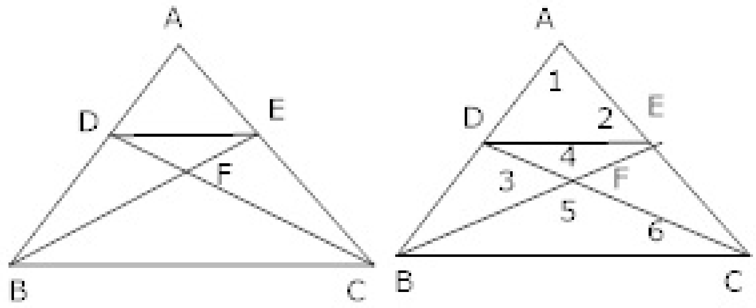
Adding (i) and (ii), we get

$$\Rightarrow \frac{BC+AB}{AC} = \frac{y}{x}+\frac{y}{z} = y\left(\frac{1}{x}+\frac{1}{z}\right)$$

$$\Rightarrow \frac{AC}{AC} = y\left(\frac{1}{x}+\frac{1}{z}\right) \Rightarrow 1 = \left(\frac{1}{x}+\frac{1}{z}\right)$$

$$\Rightarrow \frac{1}{y} = \frac{1}{x}+\frac{1}{z}$$

3. In the given figure, DE||BC and AD:DB = 5:4, find $\frac{area(\Delta DFE)}{area(\Delta CFB)}$.



Ans. In ΔADE and ΔABC,

$\angle 1 = \angle 1$ [Common]

$\angle 2 = \angle ACB$ [Corresponding $\angle s$]

$\therefore \Delta ADE \sim \Delta ABC$ [By A.A Rule]

$$\therefore \frac{DE}{BC} = \frac{AD}{AB} \text{........}(i)$$

Again in ΔDEF and ΔCFB,

$\angle 3 = \angle 6$ [Alternate $\angle s$]

$\angle 4 = \angle 5$ [Vertically opposite $\angle s$]

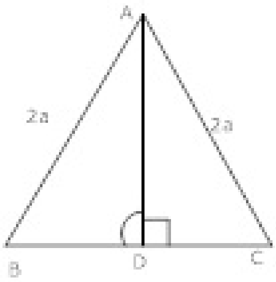
$\therefore \Delta DFE \sim \Delta CFB$ [By A.A Rule]

$$\therefore \frac{Area(\Delta DFE)}{area(\Delta CFB)} = \frac{DE^2}{BC^2} = \left(\frac{AD}{AB}\right)^2 \text{ [From (i)]}$$

$$= \left(\frac{5}{9}\right)^2 \left[\because \frac{AD}{DB} = \frac{5}{4} \Rightarrow \frac{AD}{AD+DB} = \frac{5}{5+4} \Rightarrow \frac{AD}{DB} = \frac{5}{9}\right]$$

$$\therefore \frac{area(\Delta DFE)}{area(\Delta CFB)} = \frac{25}{81}$$

4. Determine the length of an altitude of an equilateral triangle of side '2a' cm.



Ans. In right triangles ΔADB and ΔADC,

$AB = AC$

$AD = AD$

$\therefore \angle ADB = \angle ADC \quad (\text{Each} = 90°)$

$\therefore \Delta ADB \cong \Delta ADC \; (R.H.S)$

$\therefore BD = DC \; (CPCT)$

$\therefore BD = DC = a \; [\because BC = 2a]$

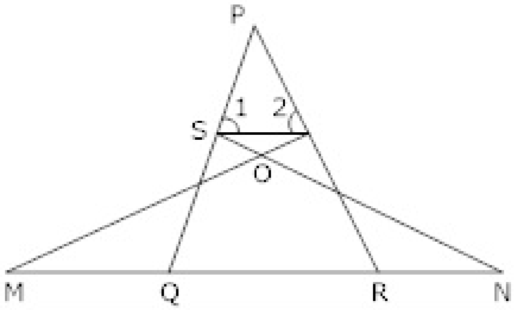
In right ΔADB, $AD^2 + BD^2 = AB^2$ (By Pythagoras Theorem)

$\Rightarrow AD^2 + a^2 = (2a)^2$

$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$

$\Rightarrow AD = \sqrt{3}a \; cm$

5. In the given figure, if $\angle 1 = \angle 2$ and $\Delta NSQ \cong \Delta MTR$. Then prove that $\Delta PTS \sim \Delta PRQ$.



Ans. Since $\Delta NSQ \cong \Delta MTR$

$\therefore \angle SQN = \angle TRM$

$\Rightarrow \angle Q = \angle R$ (in ΔPQR)

$= 90° - \frac{1}{2}\angle P$

Again $\angle 1 = \angle 2$ [given in ΔPST]

$\therefore \angle 1 = \angle 2 = \frac{1}{2}(180° - \angle P)$

$= 90° - \frac{1}{2}\angle P$

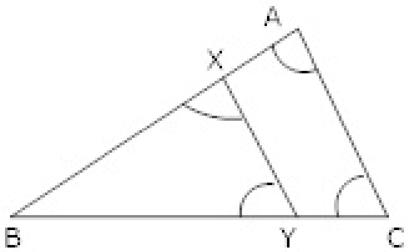
Thus, in ΔPTS and ΔPRQ

$\angle 1 = \angle Q \left[Each = 90° - \frac{1}{2}\angle P \right]$

$\angle 2 = \angle R,\ \angle P = \angle P$ (Common)

$\Delta PTS \sim \Delta PRQ$

6. In the given figure the line segment XY||AC and XY divides triangular region ABC into two points equal in area, Determine $\frac{AX}{AB}$.



Ans. Since $XY \parallel AC$

$\therefore \angle BXY = \angle BAC$

$\angle BYX = \angle BCA$

[Corresponding angles]

$\therefore \Delta BXY \sim \Delta BAC$ [A.A. similarity]

$\therefore \frac{ar(\Delta BXY)}{ar(\Delta BAC)} = \frac{BX^2}{BA^2}$

But ar$\left(\Delta BXY\right) = ar\left(XYCA\right)$

$\therefore 2\left(\Delta BXY\right) = ar\left(\Delta BXY\right) + ar\left(XYCA\right)$

$= ar\left(\Delta BAC\right)$

$\therefore \frac{ar\left(\Delta BXY\right)}{ar\left(\Delta BAC\right)} = \frac{1}{2}$

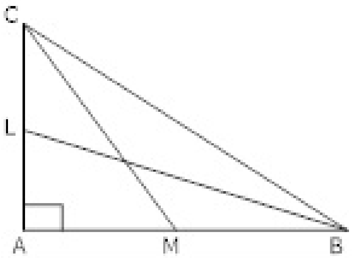
$\therefore \frac{BX^2}{BA^2} = \frac{1}{2}$

$\Rightarrow \frac{BX}{BA} = \frac{1}{\sqrt{2}}$

$\therefore \frac{BA - BX}{BA} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

7. BL and CM are medians of ΔABC right angled at A. Prove that 4(BL^2+CM^2) = 5BC^2



Ans. BL and CM are medians of a ΔABC in which $\angle A = 90°$

From ΔABC, $BC^2 = AB^2 + AC^2$.........(i)

From right angled ΔABL,

$BL^2 = AL^2 + AB^2$

i.e. , $BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$

$\Rightarrow 4BL^2 = AC^2 + 4AB^2$...........(ii)

From right-angled ΔCMA,

$CM^2 = AC^2 + AM^2$

i.e. $CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$ [Mis mid-point]

$\Rightarrow CM^2 = AC^2 + \frac{AB^2}{4}$

$\Rightarrow 4CM^2 = 4AC^2 + AB^2$.........$(iii)$

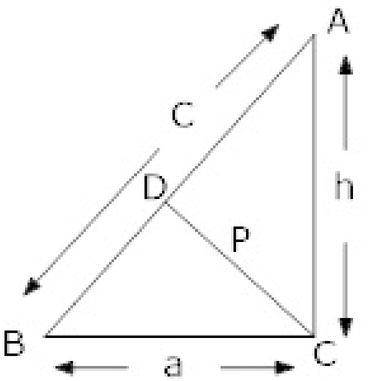
Adding (ii) and (iii), we get

i.e. $4\left(BL^2 + CM^2\right) = 5BC^2$ [From (i)]

8. ABC is a right triangle right angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB, prove that

(i) cp = ab

(ii) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



Ans. (i) Draw $CD \perp AB$

Then, $CD = p$

Now ar of $\Delta ABC = \frac{1}{2}(BC \times CA)$

$= \frac{1}{2}ab$

Also area of $\Delta ABC = \frac{1}{2}AB \times CD$

$= \frac{1}{2} cp$

Then, $\frac{1}{2} ab = \frac{1}{2} cp$

$\Rightarrow cp = ab$

(ii) Since ΔABC is a right-angled triangle with $\angle C = 90°$

$\therefore AB^2 = BC^2 + AC^2$

$\Rightarrow c^2 = a^2 + b^2$

$\Rightarrow \left(\frac{ab}{p}\right)^2 = a^2 + b^2$

$\therefore cp = ab$

$\Rightarrow c = \frac{ab}{p}$

$\Rightarrow \frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$

Thus $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. In figure, a triangle ABC is right-angled at B. side BC is trisected at points D and E, prove that 8 $AE^2 = 3AC^2 + 5AD^2$

Ans. Given: ΔABC is right-angled at B. Side BC is trisected at D and E.

To Prove: $8AE^2 = 3AC^2 + 5AD^2$

Proof: D and E are the paints of trisection of BC

$BD = \frac{1}{3} BC$ and $BE = \frac{2}{3} BC$.........(i)

In right-angled triangle ABD

$AD^2 = AB^2 + BD^2$ ....(ii) [Using Pythagoras theorem]

In ΔABE,

$AE^2 = AB^2 + BE^2$ ....(iii)

In ΔABC,

$AC^2 = AB^2 + BC^2$ ....(iv)

From (ii) and (iii), we have

$$AD^2 - AE^2 = BD^2 - BE^2$$

$$\Rightarrow AD^2 - AE^2 = \left(\frac{1}{3}BC\right)^2 - \left(\frac{2}{3}BC\right)^2$$

$$\Rightarrow AD^2 - AE^2 = \frac{1}{9}BC^2 - \frac{4}{9}BC^2 = \frac{-3}{9}BC^2$$

$$\Rightarrow AE^2 - AD^2 = \frac{1}{3}BC^2 \text{.......}(v)$$

From (iii) and (iv), we have

$$AC^2 - AE^2 = BC^2 - BE^2$$

$$= BC^2 - \frac{4}{9}BC^2$$

$$\Rightarrow AC^2 - AE^2 = \frac{5}{9}BC^2$$

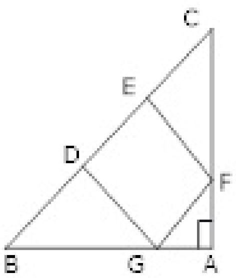
From (v) and (vi), we get

$$AC^2 - AE^2 = \frac{5}{3}\left(AE^2 - AD^2\right)$$

$$\Rightarrow 3AC^2 - 3AE^2 = 5AE^2 - 5AD^2$$

$$\Rightarrow 8AE^2 = 5AD^2 + 3AC^2$$

10. In figure, DEFG is a square and $\angle BAC = 90°$, show that $DE^2 = BD \times EC$.



Ans. Given: ΔABC is right-angled at A and DEFG is a square

To Prove: $DE^2 = BD \times EC$

Proof: Let $\angle C = x$.........(i)

Then, $\angle ABC = 90° - x$ [$\because \Delta ABC$ is right angled]

Also ΔBDG is right-angled at D.

$\angle BGD = 90° - (90° - x) = x$.......(ii)

From (i) and (ii), we get

$\angle BGD = \angle C$........(iii)

Consider ΔBDG and ΔCEF

$\angle CEF = \angle BDG = 90$ [$\because DEFG$ is square]

$\angle BGD = \angle C$ [From (iii)]

$\therefore \Delta BDG \sim \Delta FEC$ [By AA similarity]

$\therefore \frac{BD}{EF} = \frac{DG}{EC}$

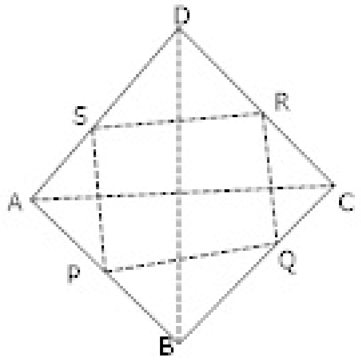
$\Rightarrow EF \times DG = BD \times EC$

But $EF = DG = DE$ [$\because$ side of a square]

$\Rightarrow DE \times DE = BD \times EC$

$\Rightarrow DE^2 = BD \times EC$

11. In a quadrilateral ABCD, P,Q,R,S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that PQRS is a parallelogram.



Ans. To Prove: PQRS is a parallelogram

Construction: Join AC

Proof: In ΔDAC,

$\frac{DS}{SA} = \frac{DR}{RC} = 1$ [∵ S and R are mid-points of AD and DC]

$\Rightarrow SR \parallel AC........(i)$ [by converse of B.P.T]

In ΔBAC, $\frac{PB}{AP} = \frac{BQ}{QC} = 1$ [∵ P and Q are mid points of AB and BC]

$\Rightarrow PQ \parallel AC........(ii)$ [By converse of B.P.T]

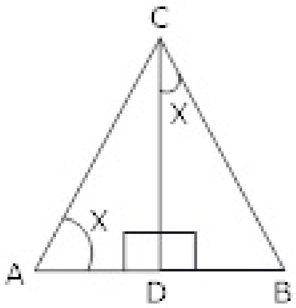
From (i) and (ii), we get

$SR \parallel PQ......(iii)$

Similarly, join B to D and PS||QR

$\Rightarrow \therefore PQRS$ is a parallelogram.

12. Triangle ABC is right-angled at C and CD is perpendicular to AB, prove that $BC^2 \times AD = AC^2 \times BD$.



Ans. Given: A ΔABC right angled at C and $CD \perp AB$

To Prove: $BC^2 \times AD = AC^2 \times BD$

Proof: Consider ΔACD and ΔDCB

Let $\angle A = x$

Then $\angle B = 90° - x$ [$\because \Delta ACB$ is right angled]

$\Rightarrow \angle DCB = x$ [$\because \Delta CDB$ is right angled]

In ΔADC and ΔCDB,

$\angle ADC = \angle CDB \left[90° \text{ each}\right]$

$\angle A = \angle DCB = x$

$\Delta ACD \sim \Delta CBD$ [By AA similarity]

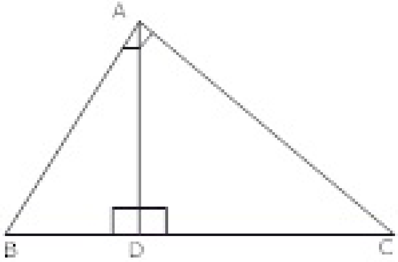
$$\Rightarrow \frac{ar\Delta ACD}{ar\Delta CBD} = \frac{AC^2}{BC^2}$$

$$\Rightarrow \frac{\frac{1}{2}AD \times CD}{\frac{1}{2}BD \times CD} = \frac{AC^2}{BC^2}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC^2}{BC^2}$$

$$\Rightarrow BC^2 \times AD = AC^2 \times BD$$

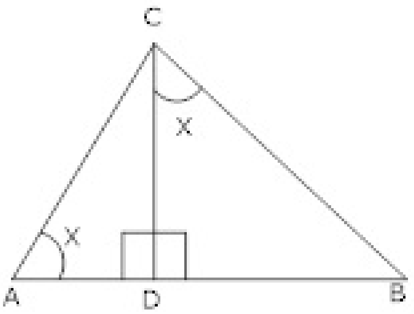
13. Triangle ABC is right angled at C and CD is perpendicular to AB. Prove that $BC^2 \times AD = AC^2 \times BD$.



Ans. Given: ΔABC right-angled at C and $CD \perp AB$

To prove: $BC^2 \times AD = AC^2 \times BD$

Proof: Consider ΔACD and ΔDCB



Let $\angle A = x$

Then $\angle B = 90 - x \left[\because \Delta ACB \text{ is right angled}\right]$

$\Rightarrow \angle DCB = x \left[\because \Delta CDB \text{ is right angled}\right]$

In ΔADC and ΔCDB,

$\angle ADC = \angle CDB \left[90^\circ \text{ each}\right]$

$\angle A = \angle DCB = x \left[\text{from above}\right]$

$\therefore \Delta ACD \sim \Delta CBD \left[\text{AA similarity}\right]$

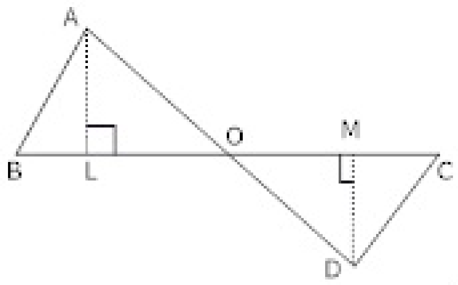
$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta VBD)} = \frac{AC^2}{BC^2}$$

$$\Rightarrow \frac{\frac{1}{2}\times AD\times CD}{\frac{1}{2}\times BD\times CD}=\frac{AC^2}{BC^2}$$

$$\Rightarrow \frac{AD}{BD}=\frac{AC^2}{BC^2}$$

$$\Rightarrow BC^2\times AD=AC^2.BD$$

14. In figure, ABC and DBC are two triangles on the same base BC. If AD intersect EC at O, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$.



Ans. Given: ABC and DBC are two triangles on the same base BC but on the opposite sides of BC, AD intersects BC at O.

Construction: Draw $AL\perp BC$ and $DM\perp BC$

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{EO}$

Proof: In ΔALO and ΔDMO,

$\angle ALO=\angle DMO\left[each\ 90^\circ\right]$

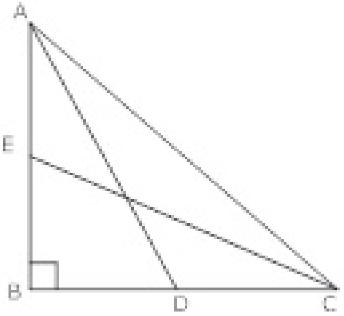
$\angle AOL=\angle DOM$ [Vertically opposite angles]

$\therefore \Delta ALO\sim\Delta DMO$ [By AA similarily]

$$\Rightarrow \frac{AL}{DM}=\frac{AO}{DO}$$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)}=\frac{AO}{DO}$$

15. In figure, ABC is a right triangle right-angled at B. Medians AD and CE are of respective lengths 5 cm and $2\sqrt{5}\ cm$, find length of AC.



Ans. Given: ΔABC with $\angle B = 90°$, AD and CE are medians

To find: Length of AC

Proof: In ΔABD right-angled at B,

$$AD^2 = AB^2 + BD^2 \text{[By pythagoras theorem]}$$

$$= AB^2 + \left(\frac{1}{2}BC\right)^2 \left[\because BD = \frac{1}{2}BC\right]$$

$$= AB^2 + \frac{1}{4}BC^2$$

$$4AD^2 = 4AB^2 + BC^2 \text{.......}(i)$$

In ΔBCE right-angled at B

$$CE^2 = BE^2 + BC^2$$

$$\Rightarrow CE^2 = \left(\frac{1}{2}AB\right)^2 + BC^2$$

$$\Rightarrow CE^2 = \frac{1}{4}AB^2 + BC^2$$

$$\Rightarrow 4CE^2 = AB^2 + 4BC^2 \text{.......}(ii)$$

$$\Rightarrow 4AD^2 + 4CE^2 = 5AB^2 + 5BC^2 = 5\left(AB^2 + BC^2\right)$$

$$\Rightarrow 4AD^2 + 4CE^2 = 5AC^2$$

Given that AD = 5 and $CE = 2\sqrt{5}$

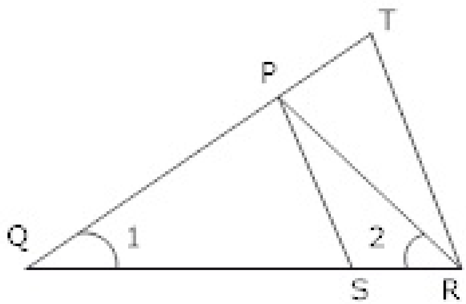
$$4(5)^2 + 4\left(2\sqrt{5}\right)^2 = 5AC^2$$

$$\Rightarrow 100 + 80 = 5AC^2$$

$$\Rightarrow AC^2 = \frac{180}{5}$$

$$\Rightarrow AC^2 = 36 \Rightarrow AC = 6cm$$

16. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$, show that $\Delta PQS \sim \Delta TQR$.



Ans. Given: $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$

Proof: As $\angle 1 = \angle 2$

$PQ = PR......(i)$ [side opposite to equal angles are equal]

Also $\frac{QR}{QS} = \frac{QT}{PR}(given).......(ii)$

$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ From (i) and (ii)

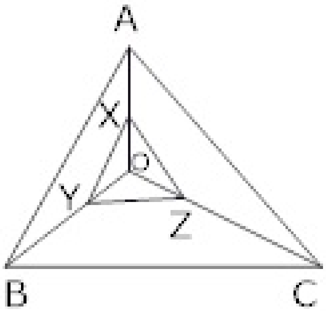
In ΔPQS and TQR, we have

$\frac{QR}{QS} = \frac{QT}{QP} = \frac{QS}{QT} \Rightarrow \frac{QR}{QP}$ [From (ii)]

Also $\angle PQS = \angle TQR$ [common]

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity]

17. Given a triangle ABC. O is any point inside the triangle ABC, X,Y,Z are points on OA, OB and OC, such that XY||AB and XZ||AC, show that YZ||AC.



Ans. Given: A $\Delta ABC, O$ is a point inside $\Delta ABC, X, Y$ *and* Z are points on OA, OB and OC respectively such that XY||AB and XZ||AB and XZ||AC

To show: YZ||BC

Proof: In $\Delta OAB, XY \parallel AB$

$$\frac{OX}{AX} = \frac{OY}{BY} \text{........}(i)[By \text{ B.P.T}]$$

In $\Delta OAC, XZ \parallel AC$

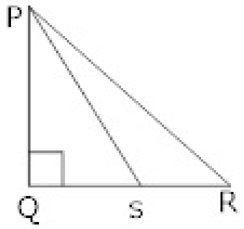
$$\therefore \frac{OX}{AX} = \frac{OZ}{CZ} \text{.........}(ii)[By \text{ B.P.T}]$$

From (i) and (ii), we get $\frac{OY}{BY} = \frac{OZ}{CZ} \text{..........}(iii)$

Now in $\Delta OBC \frac{OY}{BY} = \frac{OZ}{CZ}\left(from(iii)\right)$

$\Rightarrow YZ \parallel BC$ [Converse of B.P.T]

18. PQR is a right triangle right angled at Q. If QS = SR, show that $PR^2 = 4PS^2 - 3PQ^2$



Ans. Given: PQR is a right Triangle, right-angled at Q

Also QS = SR

To prove: $PR^2 = 4PS^2 - 3PQ^2$

Proof: In right-angled triangle PQR right angled at Q.

$PR^2 = PQ^2 + QR^2$ [By Pythagoras theorem]

Also $QS = \frac{1}{2}QR \left[\because QS = QR\right]$

In right-angled triangle PQS, right angled at Q.

$PS^2 = PQ^2 + QS^2$

$\Rightarrow PS^2 = PQ^2 + \left(\frac{1}{2}QR\right)^2 \left[From\ (ii)\right]$

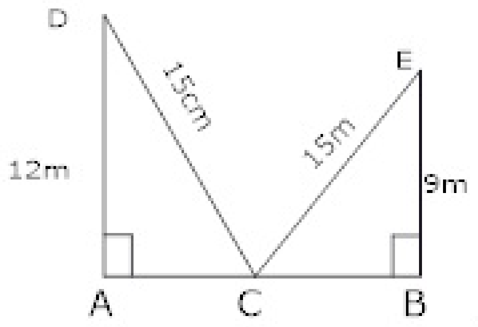
$\Rightarrow 4PS^2 = 4PQ^2 + QR^2 ....(iii)$

From (i) and (iii), we get

$PR^2 = PQ^2 + 4PS^2 - 4PQ^2$

$\Rightarrow PR^2 = 4PS^2 - 3PQ^2$

19. A ladder reaches a window which is 12 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9 m high. Find the width of the street if the length of the ladder is 15 m.



Ans. Let AB be the width of the street and C be the foot of ladder.

Let D and E be the windows at heights 12m and 9m respectively from the ground.

In ΔCAD, right angled at A, we have

$CD^2 = AC^2 + AD^2$

$\Rightarrow 15^2 = AC^2 + 12^2$

$\Rightarrow AC^2 = 225 - 144 = 81$

$\Rightarrow AC = 9\ m$

In ΔCBE, right angled at B, we have

$CE^2 = BC^2 + BE^2$

$\Rightarrow 15^2 = BC^2 + 9^2$

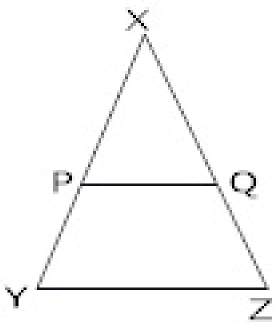
$\Rightarrow BC^2 = 225 - 81$

$\Rightarrow BC^2 = 144$

$\Rightarrow BC = 12m$

Hence, width of the street AB=AC+BC=9+12=21m

20. In figure, $\frac{XP}{PY} = \frac{XQ}{QZ} = 3$, if the area of ΔXYZ is $32\,cm^2$, then find the area of the quadrilateral PYZQ.



Ans. Given $\frac{XP}{PY}=\frac{XQ}{QZ}(given)$

$\Rightarrow PQ \parallel YZ......(i)$ [By converse of B.P.T]

In ΔXPQ and ΔXYZ, we have

[$\angle XPQ = \angle Y$ [From (i) corresponding angles]

$\angle X = \angle X$ [common]

$\therefore \Delta XPQ \sim \Delta XYZ$ [By AA similarity]

$$\therefore \frac{ar(\Delta XYZ)}{ar(\Delta XPQ)}=\frac{XY^2}{XP^2}.........(i)$$

We have $\frac{PY}{XP}=\frac{1}{3} \Rightarrow \frac{PY}{XP}+1=\frac{1}{3+1} \Rightarrow \frac{PY+XP}{XP}=\frac{4}{3}$

$$\Rightarrow \frac{XY}{XP}=\frac{4}{3}$$

Substituting in (i), we get

$$\frac{ar(\Delta XYZ)}{ar(\Delta XPQ)}=\left(\frac{4}{3}\right)^2=\frac{16}{9}$$

$$\Rightarrow \frac{32}{ar(XPQ)}=\frac{16}{9}$$

$$ar(XPQ)=\frac{32\times 9}{16}=18cm^2$$

Area of quadrilateral $PYZQ = 32-18=14cm^2$

CBSE Class 10 Mathematics

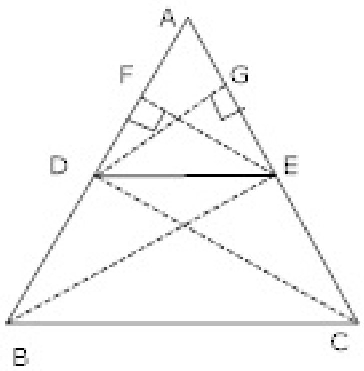
Important Questions

Chapter 6

Triangles

4 Marks Questions

1. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in district points, ten other two sides are divided in the same ratio. By using this theorem, prove that in ΔABC if $DE \parallel BC$, then $\frac{AD}{BD} = \frac{AE}{AC}$.



Ans. Given: In ΔABC $DE \parallel BC$ intersect AB at D and AC at E.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw $EF \perp AB$ and $DG \perp AC$ and join DC and BE.

Proof: $ar\Delta ADE = \frac{1}{2} AD \times EF$

$ar\Delta DBE = \frac{1}{2} DB \times EF$

$\therefore \frac{ar\Delta ADE}{ar\Delta DBE} = \frac{\frac{1}{2} AD \times EF}{\frac{1}{2} DB \times EF} = \frac{AD}{DB}$ ....(i)

Similarly, $\dfrac{ar\Delta ADE}{ar\Delta DEC}=\dfrac{\frac{1}{2}AE\times DG}{\frac{1}{2}EC\times DG}=\dfrac{AE}{EC}$.......(ii)

Since ΔDBE and ΔDEC are on the same base and between the same parallels

$\therefore ar\left(\Delta DBE\right)=ar\left(\Delta DEC\right)$

$\Rightarrow \dfrac{1}{ar\left(\Delta DBE\right)}=\dfrac{1}{ar\left(\Delta DEC\right)}$

$\therefore \dfrac{ar\Delta ADE}{ar\Delta DBF}=\dfrac{ar\Delta ADE}{ar\Delta DFC}$

$\Rightarrow \dfrac{AD}{DB}=\dfrac{AB}{EC}$

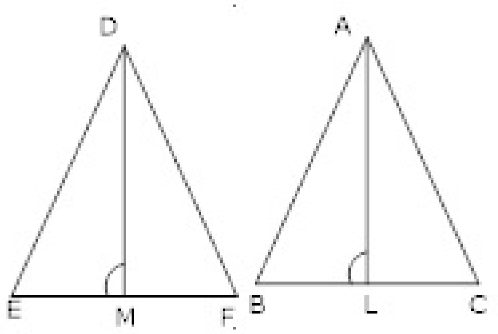
$\because DE \parallel BC$

$\dfrac{AD}{DB}=\dfrac{AE}{EC}$

$\Rightarrow \dfrac{AD}{AD+DB}=\dfrac{AE}{AE+EC}\quad \left[\because \dfrac{p}{q}=\dfrac{r}{s}\Rightarrow \dfrac{p}{p+q}=\dfrac{r}{r+s}\right]$

$\Rightarrow \dfrac{AD}{AB}=\dfrac{AE}{AC}$

2. Prove that the ratio of areas of two similar triangles are in the ratio of the squares of the corresponding sides. By using the above theorem solve in two similar triangles PQR and LMN, QR = 15cm and MN = 10 cm. Find the ratio of areas of two triangles.



Ans. Given: Two triangles ABC and DEF

Such that $\Delta ABC \sim \Delta DEF$

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction: Draw $AL \perp BC$ and $DM \perp EF$

Proof: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2}(BC)(AL)}{\frac{1}{2}(EF)(DM)}$

$\left[\because ar \text{ of } \Delta = \frac{1}{2} b \times h\right]$

$\Rightarrow \frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$ ....(i)

Again, in ΔALB and ΔDME we have

$\angle ALB = \angle DME [Each = 90^\circ]$

$\angle ABL = \angle DEM \left[\begin{array}{l}\because \Delta ABC \sim \Delta DEF \\ \therefore \angle B = \angle E\end{array}\right]$

$\therefore \Delta ALB - \Delta DME$ [By AA rule]

$$\therefore \frac{AB}{DE} = \frac{AL}{DM}$$ [∵ Corresponding sides of similar triangles are proportional]

Further, $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \text{.........}(iii)$$

From (ii) and (iii),

$$\frac{BC}{EF} = \frac{AL}{DM}$$

Putting in (i), we get

$$\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{Al}{DM} \times \frac{AL}{DM}$$

$$= \frac{AL^2}{DM^2} = \frac{AB^2}{DE^2}$$

$$= \frac{AC^2}{DF^2}$$

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Since $\Delta PQR \sim \Delta LMN$

$$\therefore \frac{ar(\Delta PQR)}{ar(\Delta LMN)} = \frac{QR^2}{MN^2} = \frac{(15)^2}{(10)^2}$$

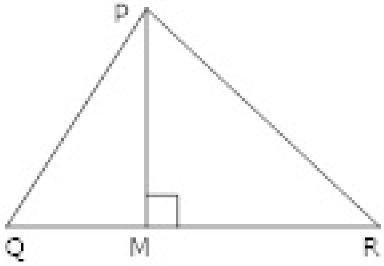
$$= \frac{225}{100} = \frac{9}{4}$$

Hence, required ratio is 9:4.

3. Prove that in a right-angled triangle the square of the hypotenuse is equal to the sum

of the squares of the other two sides.Use the above theorem in the given figure to prove that

$PR^2 = PQ^2 + QR^2 - 2QM.QR$



Ans. Given: ΔABC right-angled at A

To Prove: $BC^2 = AB^2 + AC^2$

Construction: Draw $AD \perp BC$ from A to BC

Proof: In ΔBAD and ΔABC,

$\angle B = \angle B$ [Common]

$\angle BAC = \angle BDA = 90°$

$\therefore \Delta BAD \sim \Delta BCA$ [By AA similarity]

$\therefore \frac{AB}{BC} = \frac{BD}{AB}$

$\Rightarrow AB^2 = BC \times AD$........(i)

Similarly, in ΔADC and ΔBAC

$\angle ADC = \angle BAC [90° \text{ each}]$

$\angle C = \angle C$ [Common]

$\therefore \Delta ADC \sim \Delta BAC$ [By AA similarity]

$\therefore \frac{DC}{AC} = \frac{AC}{BC}$

$\Rightarrow AC^2 = DC \times BC.......(ii)$

$(i)+(ii)$

$$AB^2 + AC^2 = BC \times BD + DC \times BC$$
$$= BC[BD + DC]$$
$$= BC \times BC$$

$\Rightarrow AB^2 + AC^2 = BC^2$

To Prove: $PR^2 = PQ^2 + QR2 - 2QM.QR$

Proof: In ΔPMR

$PR^2 = PM^2 + MR^2$ [Using above theorem]

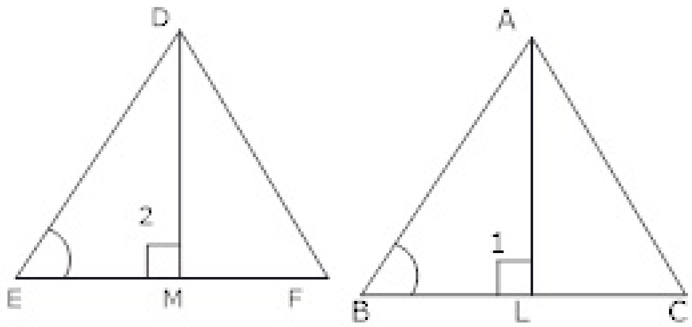
$= PM^2 + (QR - QM)^2$

$= PM^2 + QR^2 + QM^2 - 2QM.QM$

$(PM^2 + QM^2) + QR^2 - 2QM.QR$

$= PQ^2 + QR^2 - 2QM.QR\left[\because PQ^2 = QM^2 + PM^2\right]$

4. Prove that the ratio of areas of two similar triangles is equal to the square of their corresponding sides.Using the above theorem do the following the area of two similar triangles are $81\ cm^2$ and $144\ cm^2$, if the largest side of the smaller triangle is 27 cm, then find the largest side of the largest triangle.



Ans. Given: Two triangles ABC and DEF such that $\Delta ABC \sim \Delta DEF$

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction: Draw $AL \perp BC$ and $DM \perp EF$

Proof: Since similar triangles are equiangular and their corresponding sides are proportional

$\therefore \Delta ABC \sim \Delta DEF$

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$

And $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ....(i)

In ΔALB and $\Delta DMB,$

$\angle 1 = \angle 2$ and $\angle B = \angle E$

$\Rightarrow \Delta ALB \sim \Delta DME$ [By AA similarity]

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$ ....(ii)

From (i) and (ii), we get

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM}$ ....(iii)

Now $\frac{area(\Delta ABC)}{area(\Delta DEF)} = \frac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(BF \times DM)}$

$\Rightarrow \frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$

$\Rightarrow \frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$

Hence, $\frac{Area\Delta ABC}{Area\Delta DEF}=\frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}$

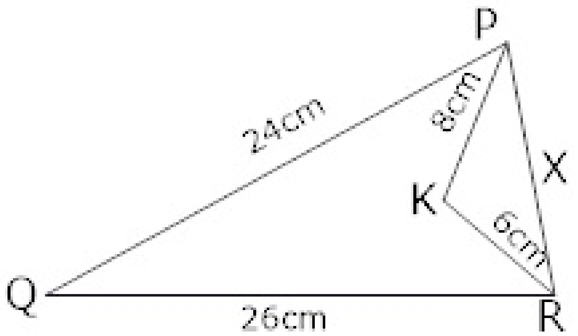
Let the largest side of the largest triangle be x cm

Using above theorem,

$$\frac{x^2}{27^2}=\frac{144}{81}\Rightarrow\frac{x}{27}=\frac{12}{9}$$

$$\Rightarrow x=36\ cm$$

5. In a triangle if the square of one side is equal to the sum of the squares on the other two sides. Prove that the angle apposite to the first side is a right angle.Use the above theorem to find the measure of $\angle PKR$ in figure given below.



Ans. Given: A ΔABC such that

$$AC^2=AB^2+BC^2$$

To prove: Triangle ABC is right angled at B

Construction: Construct a triangle DEF such that

$DE=AB, EF=BC$ andÐ $E=90^\circ$

Proof: $\because \Delta DEF$ is a right angled triangle right angled at E [construction]

$\therefore$ By Pythagoras theorem, we have

$DF^2 = DE^2 + EF^2$

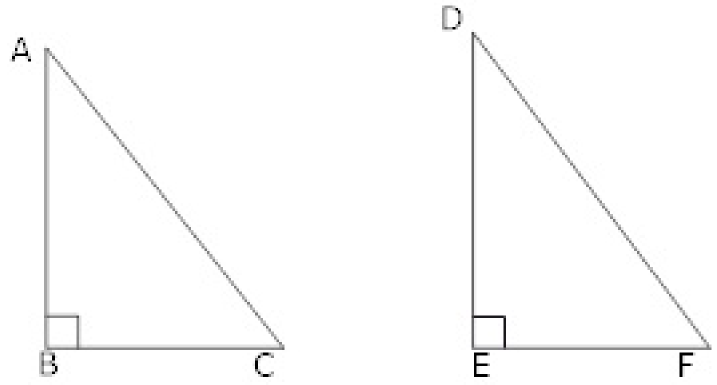
$\Rightarrow DF^2 = AB^2 + BC^2 [\because DE = AB \text{ and } EF = BC]$

$\Rightarrow DF^2 = AC^2 \left[\because AB^2 + BC^2 = AC^2\right]$

$\Rightarrow DF^2 = AC^2 \left[\because AB^2 + BC^2 = AC^2\right]$

$\Rightarrow DF = AC$

Thus, in ΔABC and ΔDEF, we have



$AB = DE, BC = EF \text{ and } AC = DF$ [By Construction and (i)]

$\therefore \Delta ABC \cong \Delta DEF$

$\Rightarrow \angle B = \angle E = 90°$

Hence, ΔABC is a right triangle.

In $\Delta QPR, \angle QPR = 90°$

$\Rightarrow 24^2 + x^2 = 26^2$

$\Rightarrow x = 10 \Rightarrow PR = 10\ cm$

Now in $\Delta PKR, PR^2 = PK^2 + KR^2 [as\ 10^2 = 8^2 + 6^2]$

$\therefore \Delta PKR$ is right angled at K

$\Rightarrow \angle PKR = 90°$